Econometrics Test
2011 - 06 - 22

Name: ____________________ Matricola: ____________________

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. Answers with no motivations will not be considered.

   (a) An invertible matrix must be square.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) If X is a scalar random variable and a and b are positive constants, then $E(a + bX^2) \geq a + bE(X)^2$
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) If $X_n \xrightarrow{\text{p}} X$, then the probability that the difference $|X_n - X|$ is small gets arbitrarily close to 1 as n gets large.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) If $X_n \xrightarrow{\text{d}} X$, then the probability that the difference $|X_n - X|$ is small gets arbitrarily close to 1 as n gets large.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) In a linear model
   $$\ln y_i = \beta_0 + \beta_1 \ln x_i + u_i$$
   the coefficient β_1 may be interpreted as the elasticity of y_i with respect to x_i.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. You're the manager of a fish restaurant in France. The number of customers who complained about the quality of the food in the past ten days is:

$$0, 0, 1, 0, 0, 0, 2, 0, 1, 0$$

assuming that these can be considered iid realisations from a random variable for which the probability function is

$$P(X = x) = e^{-m} \frac{m^x}{x!}.$$

Estimate the probability that, on a given day, at least one customer complains. You may find it useful to know that $E(x) = m$.

3. Assume that you observe, for a sample of $n = 100$ observations, the following statistics

$$\begin{aligned} y'y = \sum_{i=1}^{n} y_i^2 &= 146.4 \\ X'y = \sum_{i=1}^{n} x_i y_i &= \begin{bmatrix} -12/25 \\ 21/25 \end{bmatrix} \\ X'X = \sum_{i=1}^{n} x_i x_i' &= \begin{bmatrix} 6/25 & -3/50 \\ -3/50 & 3/8 \end{bmatrix} \end{aligned}$$

   (a) Calculate the OLS statistic $\hat{\beta}$
   (b) Calculate the sum of squared residuals $e'e$
   (c) Calculate the statistic $\hat{\sigma}^2$
   (d) Calculate the estimator of $V\left[\hat{\beta}\right]$
   (e) Test the hypothesis $\beta_1 = 0$
   (f) Test the hypothesis $\beta_2 = 0$
   (g) Test the hypothesis $\beta_1 = \beta_2$

   (Note: use the asymptotic formulae whenever possible)