Econometrics Test
2011 - 10 - 03

Name: ____________________ Matricola: ____________________

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. Answers with no motivations will not be considered.

   (a) The transpose of a singular matrix may be non-singular.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) If the covariance between the two random variables X and Y is non-zero, the two variables cannot be independent.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) If $E(X) = 0$, then $E(X \cdot Y)$ is the covariance between X and Y (provided it exists).
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) If $\sqrt{n}X_n \xrightarrow{\mathrm{d}} N(0, 1)$, then $X_n \xrightarrow{\mathrm{p}} 0$.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) If a hypothesis is rejected at the 5% significance level, it may still be accepted at the 1% significance level.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. Consider a continuous random variable X with support over the whole real line (that is, $P(a < X < b) > 0$ for any $a < b$) and with finite expectation:

$$E(X) = \alpha = \frac{1}{1 + e^{-m}}$$

where m is a real number. Suppose you have an iid sample of n such variables and that $\bar{X}$ is the sample average

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i .$$

Then:

(a) find the limit

$$\lim_{n \to \infty} P(0 < \bar{X} < 1);$$

(b) show that $\hat{m} = \ln \left(\frac{\bar{X}}{1 - \bar{X}} \right)$ is a consistent estimator of m ;

(c) assuming that $\sqrt{n}(\bar{X} - \alpha) \xrightarrow{\text{d}} N(0, \sigma^2)$, find the asymptotic distribution of $\hat{m}$;

3. For a dataset with 100 observations, you have these moment matrices:

$$\begin{aligned}
(X'X)^{-1} &= \begin{bmatrix} 0.03 & -0.006 & -0.015 \\ -0.006 & 0.01 & 0.006 \\ -0.015 & 0.006 & 0.03 \end{bmatrix} \\
X'y &= \begin{bmatrix} 100 \\ 200 \\ 50 \end{bmatrix} \\
y'y &= 2105
\end{aligned}$$

(a) Compute the OLS estimator $\hat{\beta}$

(b) Compute the estimator for the conditional variance $\hat{\sigma}^2$

(c) Compute an estimator for the covariance matrix of $\hat{\beta}$

(d) Test the hypothesis $H_0 : \beta_2 = \beta_3$