

Econometrics Test

2012 - 01 - 18

Name: _____ Matricola: _____

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. Answers with no motivations will not be considered.

(a) The matrix $A(B'A)^{-1}B'$ is symmetric.

TRUE ☐ FALSE ☐ CAN'T SAY ☐

(b) The matrix $A(B'A)^{-1}B'$ is idempotent.

TRUE ☐ FALSE ☐ CAN'T SAY ☐

(c) If $E(Y|X) = 3 + 2X$ and $E(X) = 0$, then $E(Y) = 3$.

TRUE ☐ FALSE ☐ CAN'T SAY ☐

(d) If $E(Y|X) = 3 + 2X^2$ and $E(X) = 0$, then $E(Y) \geq 3$.

TRUE ☐ FALSE ☐ CAN'T SAY ☐

(e) Suppose you estimate a model with OLS and test for a constraint of the form $R\beta = d$. The estimate of the variance s^2 for the constrained model is never smaller than s^2 for the unconstrained model.

TRUE ☐ FALSE ☐ CAN'T SAY ☐

2. Suppose you have an iid sample of random variables X_i with $E(X) = \mu$; $V(X) = 1$, and consider the statistic

$$m = 0.5 \sum_{i=1}^n (X_i / 2^{i-1})$$

So for example, if $n = 3$, m equals $0.5 \cdot (X_1 + 0.5X_2 + 0.25X_3)$.

Then:

- (a) find $E(m)$;
- (b) find $V(m)$;
- (c) find $\lim_{n \rightarrow \infty} E(m)$;
- (d) find $\lim_{n \rightarrow \infty} V(m)$;
- (e) say if m is an unbiased estimator for μ ;
- (f) say if m is a consistent estimator for μ .

Hint: you may find the results below quite useful. If \mathbf{x} is a vector of random variables, λ is a vector of constants, and α is a real number, then

$$\begin{aligned} E(\lambda' \mathbf{x}) &= \lambda' E(\mathbf{x}) \\ V(\lambda' \mathbf{x}) &= \lambda' V(\mathbf{x}) \lambda \\ \sum_{i=0}^n \alpha^i &= \frac{1 - \alpha^{n+1}}{1 - \alpha} \end{aligned}$$

3. Big Andy's Burger Barn is a fast food chain with 75 branches. Monthly sales (in US \$) are modelled as follows:

OLS, using observations 1-75
Dependent variable: sales

| | coefficient | std. error | t-ratio | p-value | |
|--------------------|-------------|--------------------|---------|----------|-----|
| const | 109.719 | 6.79905 | 16.14 | 1.87e-25 | *** |
| price | -7.64000 | 1.04594 | -7.304 | 3.24e-10 | *** |
| advert | 12.1512 | 3.55616 | 3.417 | 0.0011 | *** |
| advert^2 | -2.76796 | 0.940624 | -2.943 | 0.0044 | *** |
| Mean dependent var | 77.37467 | S.D. dependent var | | 6.488537 | |
| Sum squared resid | 1532.084 | S.E. of regression | | 4.645283 | |
| R-squared | 0.508235 | Adjusted R-squared | | 0.487456 | |
| F(3, 71) | 24.45932 | P-value(F) | | 5.60e-11 | |

where $sales_i$ is monthly sales in a given branch and is measured in \$1,000 increments, $price_i$ is price of a hamburger measured in dollars, and $advert_i$ is the advertising expenditure also measured in thousands of dollars.

Comment on the estimates, particularly on the economic meaning of the size and sign of estimated coefficients.