

Name and *matricola*: _____ email: _____

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. A "CAN'T SAY" answer with no motivations will be considered wrong. **In all exercises below, the matrices A and B are assumed square, invertible and of the same size.**

- (a) If $C = AB$, then $C' = A'B'$.

TRUE ☐ FALSE ☐ DON'T KNOW ☐

- (b) If $C = AB$, then $C' = B'A'$.

TRUE ☐ FALSE ☐ DON'T KNOW ☐

- (c) If $C = A'B$, then $(C')^{-1} = A^{-1}(B')^{-1}$

TRUE ☐ FALSE ☐ DON'T KNOW ☐

- (d) Suppose you have n random variables x_1, x_2, \dots, x_n , with $E(x_i) = m$, not necessarily iid. If $V(\bar{X}) \rightarrow 0$ as $n \rightarrow \infty$, then the sample mean \bar{X} is a consistent estimator of m .

TRUE ☐ FALSE ☐ DON'T KNOW ☐

- (e) If X_n is a sequence of random variables for which $X_n \xrightarrow{d} N(0, 22.5)$ holds, then $P(X_n < 0) \rightarrow 1/2$ as $n \rightarrow \infty$.

TRUE ☐ FALSE ☐ DON'T KNOW ☐

2. We estimated the following model via OLS on data from 75 hamburger shops:

$$\begin{aligned} \hat{s}_i &= 5.30558 - \frac{0.561675}{(0.13649)} p_i + \frac{0.0752672}{(0.022255)} a_i - \frac{0.0448928}{(0.026645)} q_i \\ T = 75 \quad \bar{R}^2 &= 0.4679 \quad F(3, 71) = 22.694 \quad \sqrt{s^2} = 0.061592 \\ &\text{(standard errors in parentheses)} \end{aligned}$$

where the variables are as follows:

Summary Statistics, using the observations 1–75

Variable	Description	Mean	Std. Dev.	Min.	Max.
s_i	Log of sales in shop i	4.34516	0.084440	4.13357	4.51305
p_i	Log of price in shop i	1.73411	0.091382	1.57485	1.87026
a_i	Log of advertising costs for shop i	0.488805	0.530873	-0.69315	1.13140
q_i	Square of a_i	0.516999	0.444210	0.00000	1.28007

Now answer the following questions:

- (a) Calculate the sum of squared residuals; $SSR =$ _____
- (b) Is β_p , the coefficient on (log) price, “statistically significant”? YES ☐ NO ☐
- (c) Test the hypothesis $\beta_p = -1$

Test statistic: _____ Distribution: _____
Decision: Accept ☐ Reject ☐

- (d) What is the expected response of sales to a 10% increase in the price of hamburgers?

response = _____

- (e) Is β_a , the coefficient on (log) advertising costs, “statistically significant”? YES ☐ NO ☐
- (f) Forecast the impact on sales for a shop that has decided to raise the advertising costs from 1.5 to 2.0 (*Hint*: $\log 1.5 \simeq 0.4$; $\log 2.0 \simeq 0.7$), keeping the price unchanged.

impact = _____

- (g) Comment on any other statistic that you consider relevant for interpreting correctly the result of the equation estimated above; if possible, indicate what you would do to improve the model and why.

3. Someone said: “In an OLS regression, the numerical values of the $\hat{\beta}$ coefficients depend on the unit of measurement of the variables involved, whereas those for the t -ratios do not”. Is this statement true? Discuss and motivate your answer. (*Hint: a t -ratio could be written as the square root of an F statistic.*)