Name: ____________________

Matricola: ____________________ email: ____________________

ECONOMETRICS - 08-06-2018 - Time: 2 h 30'

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no motivations will be considered wrong.

   (a) If A is a square matrix, x is a non-zero vector and $Ax = 0$, then A is singular.
   True ◯ False ◯ Not necessarily ◯

   (b) If $\mathbf{P_X z} = \mathbf{z}$, then $\mathbf{M_X z} = 0$.
   True ◯ False ◯ Not necessarily ◯

   (c) If $X_n \xrightarrow{\text{d}} N(0,1)$, then the probability that $X_n = 0$ tends to 1 as $n \to \infty$.
   True ◯ False ◯ Not necessarily ◯

   (d) In the OLS model, if the sum of squared residuals is 0 then $R^2 = 0$.
   True ◯ False ◯ Not necessarily ◯

   (e) Assuming you represent the restrictions to a parameter vector β as $R\beta = d$, then the matrices

$$R_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad d_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \qquad d_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   express the same restrictions.
   True ◯ False ◯ Not necessarily ◯

2. You estimated the model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \epsilon_t$$

on a cross-sectional sample with the following data:

$$(X'X)^{-1} = \frac{1}{1000} \cdot \begin{bmatrix} 5.0 & 0.0 & 0.0 \\ 0.0 & 2.5 & -1.0 \\ 0.0 & -1.0 & 2.0 \end{bmatrix} \qquad \hat{\beta} = \begin{bmatrix} 3.0 \\ 0.8 \\ -1.0 \end{bmatrix} \qquad \hat{\sigma}^2 = 2$$

(a) Compute the number of observations:[1]

$n =$ ________

(b) Compute the sum of squared residuals:

$e'e =$ ________

(c) Compute the uncentred R^2 index:

$R^2 =$ ________

(d) Perform a significance test for β_1.

Test type:	________	Distribution:	________	Test statistic:	________
Decision:	○	Reject	○	Don't reject	

(e) Perform a test for $H_0 : \beta_1 + \beta_2 = 0$

Test type:	________	Distribution:	________	Test statistic:	________
Decision:	○	Reject	○	Don't reject	

(f) Compute the RLS estimator for the hypothesis H_0 above:

$$\tilde{\beta} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

[1]Hint: the $(X'X)^{-1}$ matrix is block-diagonal.

3. Table 1 contains a dynamic regression, estimated on quarterly data for the Euro area, where the dependent variable is inflation (π_t) and the only explanatory variable is GDP growth (g_t).

   (a) Write the dynamic multipliers $\delta_j = \frac{\partial \pi_t}{\partial g_{t-j}}$ in the space below:

Multiplier	δ_0	δ_1	δ_2	δ_3	δ_4
Numerical value					

   (b) Write the long-run multiplier: $c = \sum_{i=0}^{\infty} \delta_i =$ ________

   (c) comment on sign and the magnitude of the long-run multiplier; does the sign agree with your economic intuition? Is its magnitude different from you would have expected?

   (d) comment on the Godfrey test shown in the table; does the model suffer from residual autocorrelation?

Table 1: OLS, using observations 1992:1–2016:4 ($T = 100$), dependent variable: π_t

	Coefficient	Std. Error	t-ratio	p-value
const	-0.2819	0.2229	-1.2651	0.2089
g_{t-1}	0.2692	0.0573	4.7016	0.0000
π_{t-2}	0.0957	0.0693	1.3804	0.1707
π_{t-4}	0.7882	0.0744	10.5965	0.0000

Mean dependent var	1.916673	S.D. dependent var	2.117760
Sum squared resid	161.3729	S.E. of regression	1.296521
R^2	0.636552	Adjusted R^2	0.625194
$F(3, 96)$	56.04565	P-value(F)	5.04e–21
$\hat{\rho}$	0.064485	Durbin–Watson	1.860004

LM test for autocorrelation up to order 4 –
Test statistic: LMF = 2.31863, with p-value = 0.0628467