Name: _____

Matricola: _____ email: _____

**ECONOMETRICS - 10-09-2018 - Time: 2 h 30'**

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no motivations will be considered wrong.

   (a) The identity matrix is always invertible.

   True ◯ False ◯ Not necessarily ◯

   (b) Let $X$ be a random variable whose support is the interval $[-a/2, a/2]$ and whose density function is $f(x) = \frac{1}{a}$; then $E[X] = 0$.

   True ◯ False ◯ Not necessarily ◯

   (c) Let $X$ be a random variable whose support is the interval $[-a/2, a/2]$ and whose density function is $f(x) = \frac{1}{a}$; then $E[X^2] = 0$.

   True ◯ False ◯ Not necessarily ◯

   (d) If $X_n \xrightarrow{\text{d}} X$, then $P(X_n > 0)$ is approximately equal to $P(X > 0)$ if $n$ is large.

   True ◯ False ◯ Not necessarily ◯

   (e) If the White test is rejected for a model estimated by OLS, then our estimator is inconsistent.

   True ◯ False ◯ Not necessarily ◯

2. Suppose you have the following model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \varepsilon_i;$$

you estimate it on a sample of 200 observations under different restrictions, and these are the results you get:

| **Restriction** | **Sum of squared residuals** |
|---|---|
| None | 211.202 |
| $\beta_1 = 1$ | 212.612 |
| $\beta_2 = 1$ | 214.281 |
| $\beta_1 = \beta_2$ | 215.360 |
| $\beta_1 = \beta_2 = 1$ | 215.555 |

(a) Test the hypothesis $\beta_1 = 1$;

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(b) Test the hypothesis $\beta_2 = 1$;

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(c) Test the hypothesis $\beta_1 = \beta_2$;

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(d) Test the hypothesis $\beta_1 = \beta_2 = 1$;

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(e) Are there any contradictions among the results of your tests? Discuss in the space below

3. The following model was estimated on a sample of 1000 US workers:

$$w_i = \beta_0 + \beta_1 g_i + \beta_2 e_i + \beta_3 x_i + \beta_4 e_i^2 + \beta_5 e_i \cdot x_i + \beta_6 x_i^2 + \varepsilon_i$$

where

- $w_i$ is the wage for individual $i$ in US$;
- $g_i$ is a dummy variable: 1 for female, 0 for male;
- $e_i$ is the individual's education, in years;
- $x_i$ is the individual's work experience, in years.

The estimates are as follows:

Dependent variable: $w_i$
Heteroskedasticity-robust standard errors, variant HC1

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| $\beta_0$ | 1.4171 | 6.9880 | 0.2028 | 0.8393 |
| $\beta_1$ | −4.1608 | 0.7164 | −5.8078 | 0.0000 |
| $\beta_2$ | −0.4985 | 0.7769 | −0.6417 | 0.5212 |
| $\beta_3$ | 0.6747 | 0.2211 | 3.0514 | 0.0023 |
| $\beta_4$ | 0.0910 | 0.0239 | 3.8043 | 0.0002 |
| $\beta_5$ | 0.0047 | 0.0128 | 0.3673 | 0.7135 |
| $\beta_6$ | −0.0108 | 0.0017 | −6.3724 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 20.61566 | S.D. dependent var | 12.83472 |
| Sum squared resid | 120959.8 | S.E. of regression | 11.03687 |
| $R^2$ | 0.264974 | Adjusted $R^2$ | 0.260533 |
| $F(6, 993)$ | 55.99116 | P-value($F$) | 1.17e–59 |

(a) Is it legitimate to say that female workers get lower wages than male workers with the same education and experience? Motivate your answer.

(b) Estimate the wage for a female worker with 10 years of experience and 13 years of education.

$$\hat{w} = \quad \_\_\_\_\_$$

(c) Estimate the marginal effect of education on wage for a female worker with 10 years of experience and 13 years of education.

$$\frac{\partial w_i}{\partial e_i} = \quad \_\_\_\_\_$$

(d) Estimate the marginal effect of experience on wage for a female worker with 10 years of experience and 13 years of education.

$$\frac{\partial w_i}{\partial x_i} = \quad \_\_\_\_\_$$