Name: _____

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ECONOMETRICS - 30/05/2019 - Time: 2 h

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no motivations will be considered wrong.

   (a) Any vector with two elements can be written as a linear combination of the vectors $\mathbf{x}_1 = [2, 1]$ and $\mathbf{x}_2 = [2, 0]$.

   True ◯ False ◯ Not necessarily ◯

   (b) Let X_n a sequence of random variables with $E(X_n) = 1$ and $V(X_n) = \frac{1}{\sqrt{n}}$; then, $X_n \xrightarrow{\text{p}} 0$.

   True ◯ False ◯ Not necessarily ◯

   (c) A covariance matrix may be singular.

   True ◯ False ◯ Not necessarily ◯

   (d) In a linear regression model, the uncentred version of the R^2 index cannot be smaller that the centred version.

   True ◯ False ◯ Not necessarily ◯

   (e) When testing hypotheses in a linear model, the t statistic may be negative, but the corresponding Wald test statistic is always positive.

   True ◯ False ◯ Not necessarily ◯

2. Suppose you have a consistent estimator for the vector of parameters a and b:

$$\begin{aligned} \hat{\theta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} &\xrightarrow{\text{p}} \theta = \begin{bmatrix} a \\ b \end{bmatrix} \\ \sqrt{n}\left[\hat{\theta} - \theta\right] &\xrightarrow{\text{d}} N\left[0, V\right] \\ V &= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \end{aligned}$$

(a) Find a consistent estimator of the ratio $\kappa = a/b$:

$\hat{\kappa} =$ _____

(b) Find the asymptotic variance of $\hat{\kappa}$:

$\sqrt{n}(\hat{\kappa} - \kappa) \xrightarrow{\text{d}} N(0, \omega)$, where $\omega =$ _____

Now assume that $n = 180$, $\hat{a} = 2.9$, $\hat{b} = 3.1$, and a consistent estimate of ρ equals $\hat{\rho} = 0.8$.

(c) Compute the numerical values for the estimated covariance matrix of $\hat{\theta}$:

$$\hat{V} = \begin{bmatrix} & \\ & \end{bmatrix}$$

(d) Test the hypothesis $a = 3$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(e) Test the hypothesis $b = 3$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(f) Test the hypothesis $a = b$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(g) Test the hypothesis $\kappa = 1$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

Hint: all test can be computed via the $R\theta = d$ approach.

3. We have a sample of 2724 Belgian households; we build a model for the share of tobacco on the overall household expenditure, s_i,[1] as follows:

$$s_i = \beta_0 + \beta_1 x_i + \beta_2 y_i + \beta_3 a_i + \varepsilon_i, \tag{1}$$

where x_i is the number of adults in the household, y_i is the household total income (in logarithm) and a_i is a linear function of the age of the household head[2].

Table 1: Descriptive statistics

Summary Statistics, using the observations 1–2724

Variable	Mean	Median	S.D.	Min	Max
s	1.2243	0.00	2.4919	0.00	19.276
x	1.97	2.00	0.805	1.00	7.00
y	13.7	13.8	0.482	11.8	15.3
a	2.41	2.00	1.37	0.00	4.00

Summary statistics for the variables are displayed in Table 1. In order to control for possible nonlinearities, an alternative version of the model was also estimated:

$$s_i = \beta_0 + \beta_1 x_i + \beta_2 y_i + \beta_3 a_i + \beta_4 y_i^2 + \beta_5 y_i \cdot a_i + \beta_6 a_i^2 + \varepsilon_i \tag{2}$$

Answer the following questions, on the basis of the robust OLS estimates reported in Table 2.

(a) Would you prefer model (1) or model (2)? Motivate your answer on the grounds of the model reduction test and the RESET tests.

(b) What is the effect of income on tobacco smoking? Is the effect you find conform to your economic intuition?[3]

[1] The share is expressed in percentage points.
[2] Age is measured in 10-year interval classes ranging from 0 (younger than 30) to 4 (60 or older).
[3] Hint: ask yourself is tobacco is an inferior, normal, or superior good. Think about its Engel curve.

(c) Predict the share of tobacco for a household with 2 adults, whose log income is 13 and whose head is 40 years old, using the estimates for model (1).

$$\hat{s}_i = \quad \_\_\_\_\_$$

(d) Predict the share of tobacco for a household with 2 adults, whose log income is 13 and whose head is 40 years old, using the estimates for model (2).

$$\hat{s}_i = \quad \_\_\_\_\_$$

(e) Calculate the marginal effect of log income for a household whose log income is 13 and whose head is 40 years old, using the estimates for model (1).

$$\frac{\partial E(s_i|x_i, y_i, a_i)}{\partial y_i} = \quad \_\_\_\_\_$$

(f) Calculate the marginal effect of log income for a household whose log income is 13 and whose head is 40 years old, using the estimates for model (2).

$$\frac{\partial E(s_i|x_i, y_i, a_i)}{\partial y_i} = \quad \_\_\_\_\_$$

Table 2: OLS estimates

	(1)	(2)
const	20.21**	102.4**
	(10.81)	(3.015)
x	0.2746**	0.1977**
	(4.248)	(3.064)
y	−1.376**	−12.39**
	(−10.15)	(−2.552)
a	−0.2664**	−5.588**
	(−7.502)	(−4.707)
y^2		0.3635**
		(2.093)
$y \cdot a$		0.4415**
		(5.246)
a^2		−0.1510**
		(−3.911)
n	2724	2724
$\bar{R}^2$	0.0649	0.0845
ℓ	6286	6316
RESET p-value	9.22e−07	0.218

robust t-statistics in parentheses

Robust test for model reduction (2) → (1):
Test statistic: Robust F(3, 2717) = 15.342, p-value 6.73524e-10