Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 01-09-2021 - Time: 2 h 30'

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no adequate motivation will be considered wrong.

   (a) Let A and B be $n \times n$ matrices, and x an $n \times 1$ vector. Then, $Ax = Bx$ implies $A = B$.
   True ◯ False ◯ Not necessarily ◯

   (b) If $P(A|B) = P(A)$, then the events A and B are independent.
   True ◯ False ◯ Not necessarily ◯

   (c) You have an $n \times 1$ vector $\mathbf{y}$ and an $n \times k$ matrix $\mathbf{X}$. If $\mathbf{M_X y} = 0$, then the R^2 index for the regression of $\mathbf{y}$ on $\mathbf{X}$ is 1.
   True ◯ False ◯ Not necessarily ◯

   (d) Suppose that x_i is a random variable whose support is $(1, \infty)$, and that $E(y_i|x_i) = 0.5x_i^2 - x_i$. Then, the marginal effect of x_i on y_i is always positive.
   True ◯ False ◯ Not necessarily ◯

   (e) Heteroskedasticity makes the OLS estimator inconsistent.
   True ◯ False ◯ Not necessarily ◯

2. By using a CAN estimator, you have estimated a (3×1) parameter vector θ as

$$\hat{\theta} = [0.5 \quad 1 \quad 2]'$$

and the associated estimated covariance matrix is

$$\hat{V} = \begin{bmatrix} 0.04 & 0.08 & 0.12 \\ 0.08 & 0.36 & 0.24 \\ 0.12 & 0.24 & 0.48 \end{bmatrix}.$$

Test the following hypotheses:

(a) $H_0 : \theta_1 = 0$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(b) $H_0 : \theta_2 = 0$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(c) $H_0 : \theta_1 = \theta_2$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(d) $H_0 : \begin{cases} \theta_1 = \theta_2 \\ \theta_1 = \theta_3 \end{cases}$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(e) $H_0 : \theta_1 \cdot \theta_3 = \theta_2$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

3. Figure 1 depicts weekly data for the second wave of the COVID pandemic in Italy: number of deaths and average number of patients in intensive care units. Table 1 contains the OLS estimates of an ECM model.

Answer the following questions, using the numerical estimates that you find in table 1:

(a) Do we have to worry about autocorrelation here? Why?

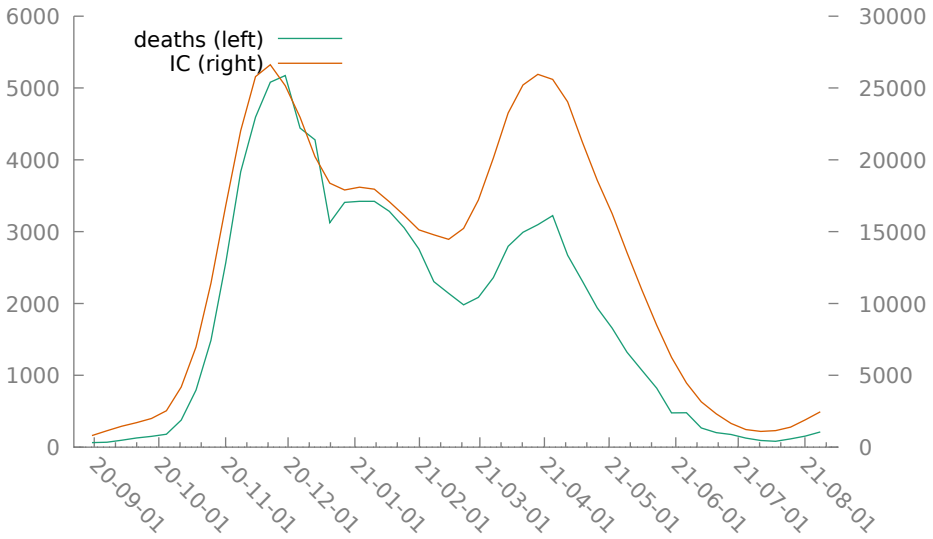

Figure 1: Weekly COVID data for Italy (2nd wave)

OLS, using observations 2020-08-31–2021-08-09 ($T = 50$)
Dependent variable: Δ log deaths

	Coefficient	Std. Error	t-ratio	p-value
const	−0.7335	0.3566	−2.0571	0.0455
time	−0.0012	0.0016	−0.7502	0.4570
Δ log IC	1.1192	0.1037	10.7882	0.0000
log deaths(-1)	−0.1799	0.0807	−2.2295	0.0308
log IC(-1)	0.2255	0.0993	2.2701	0.0280

Mean dependent var	0.033354	S.D. dependent var	0.292867
Sum squared resid	0.661068	S.E. of regression	0.121204
R^2	0.842707	Adjusted R^2	0.828725
$F(4, 45)$	60.27250	P-value(F)	1.68e–17
$\hat{\rho}$	−0.083286	Durbin–Watson	2.160428

LM test for autocorrelation up to order 4: Test statistic: = 1.8042 (p-value = 0.146527)

Table 1: Dynamic model for the COVID data

(b) Compute the first 3 dynamic multipliers:

$\delta_0 =$ _____ $\delta_1 =$ _____ $\delta_2 =$ _____

(c) The following is the estimate of the covariance matrix for the OLS model shown in Table 1:

$$\hat{V} = \frac{1}{10000} \times \begin{bmatrix} 1271.5 & -0.52732 & -68.208 & 245.80 & -325.17 \\ -0.52732 & 0.025658 & 1.0724 & 0.36207 & -0.37057 \\ -68.208 & 1.0724 & 107.63 & 12.948 & -8.8943 \\ 245.80 & 0.36207 & 12.948 & 65.086 & -79.094 \\ -325.17 & -0.37057 & -8.8943 & -79.094 & 98.642 \end{bmatrix}.$$

Test the hypothesis $H_0 : \beta_4 + \beta_5 = 0$

Test type: _____ Distribution: _____ Test statistic: _____

Decision: ◯ Reject ◯ Don't reject

(d) Comment on the result of the hypothesis test above and its implication for the long-run multiplier.