Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 2022-06-06 - Time: 2 h 30'

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no adequate motivation will be considered wrong.

   (a) The transpose of the identity matrix is equal to its inverse.

   True ◯ False ◯ Not necessarily ◯

   (b) If the support of a random variable X is only one point, then $V(X) = 0$.

   True ◯ False ◯ Not necessarily ◯

   (c) Suppose that $E(Y|X) = 1 - \log(X)$, then the elasticity of Y with respect to X is constant.

   True ◯ False ◯ Not necessarily ◯

   (d) The Chow test cannot be applied to dynamic models.

   True ◯ False ◯ Not necessarily ◯

   (e) The Godfrey test cannot be applied to models estimated using *cross-section* samples.

   True ◯ False ◯ Not necessarily ◯

2. For the model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

you have the following data:

$$X'X = \begin{bmatrix} 120 & 120 \\ 120 & 240 \end{bmatrix} \qquad X'y = \begin{bmatrix} 300 \\ 240 \end{bmatrix} \qquad y'y = 1080$$

(a) Compute the OLS estimates: $\hat{\beta}_0 =$ _____ $\hat{\beta}_1 =$ _____

(b) Compute the estimate of the variance: $\hat{\sigma}^2 =$ _____

(c) Compute the predicted value for y_i when $x_i = 2$: $\hat{y}_i =$ _____

(d) Compute the 95% confidence interval for $\hat{y}_i$:

$$y_i \in [\ \_\_\_\_\_\ ,\ \_\_\_\_\_\ ]$$

(e) Test the hypothesis $H_0 : \beta_1 = 0$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(f) Test the hypothesis $H_0 : \beta_0 = 8\beta_1$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

3. In a 1978 paper[1] the hedonic model reported in Table 1 was presented. The model was estimated using 508 towns of the *Boston Standard Metropolitan Statistical Area*. The variables used for the estimation are described in the following table:

Variable	Description	Mean	Median	SQM	Min	Max
MV	Median value of owner-occupied homes in $1000's	22.5328	21.2	9.1971	5	50
RM	average number of rooms per dwelling	6.28463	6.2085	0.702617	3.561	8.78
AGE	proportion of owner-occupied units built prior to 1940	68.5749	77.5	28.1489	2.9	100
DIS	weighted distances to five Boston employment centres	3.79504	3.20745	2.10571	1.1296	12.1265
RAD	index of accessibility to radial highways	9.54941	5	8.70726	1	24
TAX	full-value property-tax rate per $10,000 [$/10k]	408.237	330	168.537	187	711
PTRATIO	pupil-teacher ratio by town	18.4555	19.05	2.16495	12.6	22
B	proportion of blacks by town	356.674	391.44	91.2949	0.32	396.9
LSTAT	% lower status of the population	12.6531	11.36	7.14106	1.73	37.97
CRIM	per capita crime rate by town	3.61352	0.25651	8.60155	0.00632	88.9762
ZN	proportion of residential land zoned for lots over 25,000 sq.ft.	11.3636	0	23.3225	0	100
INDUS	proportion of non-retail business acres per town	11.1368	9.69	6.86035	0.46	27.74
CHAS	Charles River dummy variable (1 if tract bounds river; 0 otherwise)	0.06917	0	0.253994	0	1
NOX	nitric oxides concentration (parts per 10 million) [parts/10M]	0.554695	0.538	0.115878	0.385	0.871

Answer the following questions:

[1] Harrison and Rubinfeld (1978), "Hedonic housing prices and the demand for clean air", **Journal of Environmental Economics and Management**, Vol 5(1)

(a) Comment on two or three values of the estimated coefficients of your choosing, saying whether their sign and magnitude correspond to your economic intuition .

(b) Discuss the results of the diagnostic tests

(c) Test the hypothesis that the coefficient associated to the variable $\texttt{NOX}^2$ is equal to -1.

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

Table 1: A model for house prices

Dependent variable: log(MV)

	Coefficient	Std. Error	t-ratio	p-value
const	4.5578	0.1544	29.5116	0.0000
RM^2	0.0063	0.0013	4.8226	0.0000
AGE	0.0001	0.0005	0.1724	0.8632
log(DIS)	−0.1913	0.0334	−5.7275	0.0000
log(RAD)	0.0957	0.0191	5.0021	0.0000
TAX	−0.0004	0.0001	−3.4261	0.0007
PTRATIO	−0.0311	0.0050	−6.2081	0.0000
B	0.0004	0.0001	3.5271	0.0005
log(LSTAT)	−0.3712	0.0250	−14.8406	0.0000
CRIM	−0.0119	0.0012	−9.5320	0.0000
ZN	0.0001	0.0005	0.1585	0.8741
INDUS	0.0002	0.0024	0.1013	0.9193
CHAS	0.0914	0.0332	2.7527	0.0061
NOX^2	−0.6380	0.1131	−5.6393	0.0000

Mean dependent var	3.034513	S.D. dependent var	0.408757
Sum squared resid	16.37823	S.E. of regression	0.182453
R^2	0.805891	Adjusted R^2	0.800762
$F(13, 492)$	157.1277	P-value(F)	1.2e–165

White's test for heteroskedasticity – Test statistic: LM = 203.873
with p-value = $P(\chi^2(103) > 203.873)$ = 1.26982e-08

RESET test for specification (squares only) – Test statistic: $F(1, 491) = 4.59999$
with p-value = 0.0324622

RESET test for specification –Test statistic: $F(2, 490) = 2.40279$
with p-value = 0.09153