Name: _____

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**ECONOMETRICS - 15-02-2022 - Time: 2 h 30'**

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no adequate motivation will be considered wrong.

   (a) If $A$ is an $n \times n$ matrix and $\mathbf{b}$ is an $n \times 1$ vector, then $A \cdot \mathbf{b} \cdot \mathbf{b}'$ is an $n \times n$ matrix.
   True ◯ False ◯ Not necessarily ◯

   (b) Suppose that $X_n \xrightarrow{\text{p}} X$; then $\lim_{n\to\infty} P(X_n > 0) = P(X > 0)$.
   True ◯ False ◯ Not necessarily ◯

   (c) Suppose that you are estimating two parameters $\theta_1$ and $\theta_2$, and two separate hypothesis tests for $\theta_1 = 0$ and $\theta_2 = 0$ both accept the null. Then we can conclude that $\theta_1 = \theta_2$.
   True ◯ False ◯ Not necessarily ◯

   (d) In a linear regression model, the $R^2$ index can never be exactly 1.
   True ◯ False ◯ Not necessarily ◯

   (e) In the model $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i \cdot d_i + \varepsilon_i$, the marginal effect of $x_i$ on $y_i$ is constant.
   True ◯ False ◯ Not necessarily ◯

2. Consider two alternative models, both estimated on a dataset containing $n = 256$ observations:

$$\hat{y}_i = 10 + 0.8x_i \qquad SSR = 512 \tag{1}$$
$$\hat{y}_i = 12 + x_i - 0.1x_i^2 \qquad SSR = 480 \tag{2}$$

Suppose also that $\sum_{i=1}^{n}(y_i - \bar{Y})^2 = 1024$. Then,

(a) Compute an estimate of the conditional variance for both models:

$\hat{\sigma}_1^2 =$ _____ $\hat{\sigma}_2^2 =$ _____

(b) Compute the centred $R^2$ index for both models:

$R_1^2 =$ _____ $R_2^2 =$ _____

(c) Perform a test for the equivalence of models (1) and (2).

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

quad

(d) Using the result of the previous question, calculate an estimate of the marginal effect of $x$ on $y$, for $x_i = 2$:

$$\left.\frac{\partial E(y_i|x_i)}{\partial x_i}\right|_{x_i=2} = \quad \text{_____}$$

3. According to the so-called "Okun's Law", there is an inverse relationship between the unemployment rate and the cyclical component of GDP. In the gretl dataset `okun.gdt`, these variables for the US economy are called `unrate` ($u_t$) and `ogap` ($y_t$),[1] respectively.

Figure 1 describes the time path of these two series for the US economy between 1948 and 2021 (quarterly data). An ECM model was estimated, and the results are contained in Table 1.

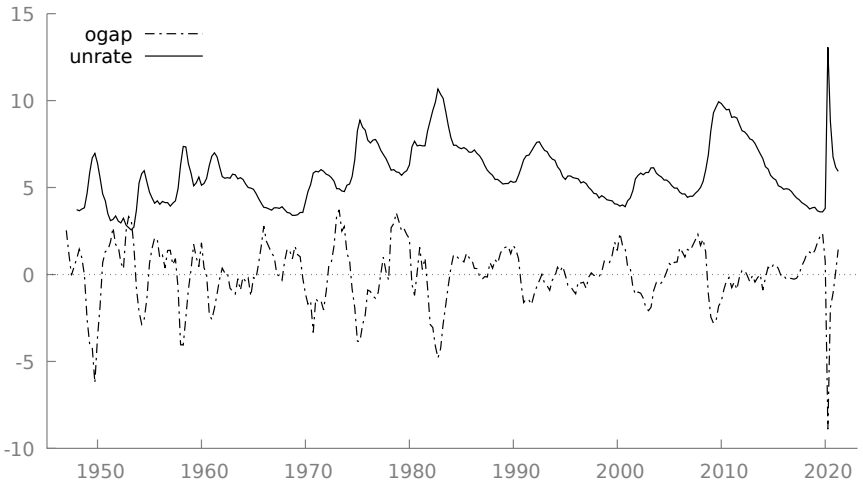


Figure 1: The $y_t$ and $u_t$ series

[1] Technically, $y_t$ was computed by applying the Hodrick-Prescott filter to the log of real GDP.

Table 1: Okun's law Model

OLS, using observations 1948:2–2021:2 ($T = 293$). Dependent variable: $\Delta u_t$

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 0.3246 | 0.1078 | 3.01 | 0.0028 |
| $\Delta y_t$ | −0.5275 | 0.0233 | −22.60 | 0.0000 |
| $\Delta y_{t-1}$ | −0.0587 | 0.0237 | −2.48 | 0.0137 |
| $u_{t-1}$ | −0.0551 | 0.0182 | −3.02 | 0.0028 |
| $y_{t-1}$ | −0.0856 | 0.0202 | −4.25 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.007509 | S.D. dependent var | 0.717378 |
| Sum squared resid | 49.27720 | S.E. of regression | 0.413644 |
| $R^2$ | 0.672081 | Adjusted $R^2$ | 0.667526 |
| $F(4, 288)$ | 147.5663 | P-value($F$) | 1.82e–68 |
| Log-likelihood | −154.5818 | Akaike criterion | 319.1636 |
| Schwarz criterion | 337.5645 | Hannan–Quinn | 326.5335 |
| $\hat{\rho}$ | −0.097695 | Durbin–Watson | 2.187151 |

LM test for autocorrelation up to order 4: LMF = 2.23577, $p$-value = 0.0653307

(a) Comment on the result from Godfrey's test:

(b) Rewrite the model in ADL form:

$$u_t = \_\_\_\_\_ + \_\_\_\_\_ u_{t-1} +$$
$$+ \_\_\_\_\_ y_t + \_\_\_\_\_ y_{t-1} + \_\_\_\_\_ y_{t-2} + \varepsilon_t$$

(c) Compute the long-run multiplier:

$$LRM = \_\_\_\_\_$$

(d) Comment on the sign and magnitude of the long-run multiplier; is it consistent with Okun's law?:

(e) Which line in Figure 2 corresponds to the sequence of cumulated multipliers for the model you estimated?

A ◯ B ◯ C ◯

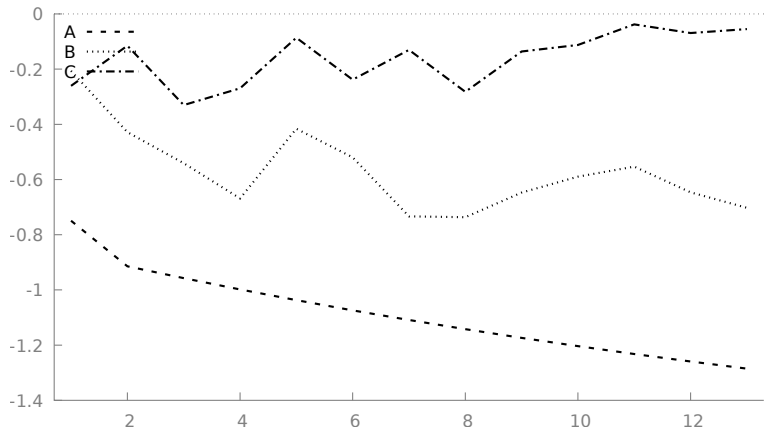


Figure 2: Three possible multiplier sequences