Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 01-04-2022 - Time: 2 h 30'

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no adequate motivation will be considered wrong.

   (a) If $\boldsymbol{\iota}$ is a vector full of ones and A is a square matrix, then $\boldsymbol{\iota}'A\boldsymbol{\iota}$ yields the sum of the elements of A.

   True ◯ False ◯ Not necessarily ◯

   (b) Let $\hat{\theta}$ and $\tilde{\theta}$ be two CAN estimators of the same parameter θ. Then $\hat{\theta} - \tilde{\theta} \xrightarrow{\text{p}} 0$.

   True ◯ False ◯ Not necessarily ◯

   (c) Let $\hat{\theta}$ and $\tilde{\theta}$ be two CAN estimators of the same parameter θ. Then $V(\hat{\theta} - \tilde{\theta}) = 0$.

   True ◯ False ◯ Not necessarily ◯

   (d) Suppose you have two linear models A and B, where B is a special case of A. Then, the R^2 index for B is always larger than that for A.

   True ◯ False ◯ Not necessarily ◯

   (e) Homoskedasticity is a necessary condition for the consistency of the OLS estimator.

   True ◯ False ◯ Not necessarily ◯

2. Using a dataset with 120 observations, you estimated the model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \varepsilon_i$$

under different restrictions, as follows:

$$\hat{y}_i = 4.9 + 1.25x_i + 0.46z_i \qquad SSR = 336 \tag{1}$$
$$\hat{y}_i = 5.03 + 0.72(x_i + z_i) \qquad SSR = 342 \tag{2}$$
$$\hat{y}_i = 5.0 + 1.7x_i \qquad SSR = 348 \tag{3}$$
$$\hat{y}_i = 5.3 + 0.9z_i \qquad SSR = 366 \tag{4}$$
$$\hat{y}_i = 5.8 \qquad SSR = 432 \tag{5}$$

Test the following hypotheses:

(a) $H_0 : \beta_1 = 0$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(b) $H_0 : \beta_2 = 0$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(c) $H_0 : \beta_1 = \beta_2$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(d) $H_0 : \begin{cases} \beta_1 = \beta_2 \\ \beta_2 = 0 \end{cases}$

Test type: _____ Distribution: _____ Test statistic: _____
Decision: ◯ Reject ◯ Don't reject

(e) Compute an estimate of the variance σ^2 using the model that you consider the best among (1) — (5) (indicate which one):

Model _____ $\hat{\sigma}^2 =$ _____

3. An experiment was carried out in Tennessee between 1985 and 1989. A group of 5752 students was followed from kindergarten through third grade. In the experiment children were randomly assigned within schools into two types of classes: small classes with 13–17 and regular-sized classes with 22–25 students. A description of the available variables is displayed in table 1.

Using the estimates provided in Table 2, answer the following questions:

(a) Comment on the R^2 statistic: would you say that the model fits the data well?

Table 1: Descriptive statistics

Variabile	Descrizione	Mean	Median	StDev	Min	Max
`math`	proficiency score in mathematics (in logs)	6.181	6.182	0.09714	5.768	6.439
`boy`	dummy, 1 if child is male	0.5135	1.000	0.4999	0.000	1.000
`small`	dummy, 1 if child is in a small class	0.6535	1.000	0.4759	0.000	1.000
`te`	teacher years of experience	9.306	9.000	5.768	0.000	27.00
`freelunch`	child is entitled to free lunch	0.4817	0.000	0.4997	0.000	1.000
`tchwhite`	dummy, 1 if the teacher is white	0.8355	1.000	0.3708	0.000	1.000
`black`	dummy, 1 if the child is black	0.3209	0.000	0.4669	0.000	1.000
`both_white`	dummy, 1 if both the teacher and the child are white	0.6376	1.000	0.4807	0.000	1.000

(b) Do we have to worry about heteroskedasticity here? Why?

(c) Comment on the sign and magnitude of the coefficients for the variables `boy, small` and `freelunch`:

(d) It is sometimes claimed that white and Asian children are better at maths than black kids. Do the estimates support this view?

(e) What is the effect of teacher experience on the children's proficiency?

(f) Is any interesting effect related to the ethnicity of the teacher?

Table 2: Model for math proficiency

OLS, using observations 1–5752
Dependent variable: math
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	6.1987	0.0078	795.9582	0.0000
boy	−0.0161	0.0024	−6.5740	0.0000
small	0.0085	0.0026	3.3019	0.0010
te	−0.0017	0.0007	−2.5967	0.0094
te2	0.0001	0.0000	4.3401	0.0000
freelunch	−0.0403	0.0027	−14.9059	0.0000
tchwhite	−0.0267	0.0046	−5.8013	0.0000
black	0.0070	0.0079	0.8971	0.3697
both_white	0.0377	0.0083	4.5211	0.0000

Mean dependent var	6.180815	S.D. dependent var	0.097179
Sum squared resid	49.36392	S.E. of regression	0.092712
R^2	0.091096	Adjusted R^2	0.089829
$F(8, 5743)$	71.94948	P-value(F)	2.3e–113
Log-likelihood	5522.512	Akaike criterion	−11027.02
Schwarz criterion	−10967.11	Hannan–Quinn	−11006.17

White's test: Test statistic = 103.006, p-value = 7.20754e-09